STANDARD COSMOLOGICAL MODEL

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The Standard Cosmological Model

- The Standard Cosmological Model is:

the model of expanding Universe with flat hypersurface which is filled by different types of matter: small amount of relativistic matter (photons), baryonic matter and dark matter which also obey dust like equation of state, and dark energy or quintessense which obeys almost vacuum dominated equation of state.

WMAP

Old Universe – *New* Numbers

$\Omega_{tot} = 1.02^{+0.02}_{-0.02}$

$w < -0.78$ (95% CL)

$\Omega_\Lambda = 0.73^{+0.04}_{-0.04}$

$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$

$\Omega_b = 0.044^{+0.004}_{-0.004}$

$n_b = 2.5 \times 10^{-7}{}^{+0.1\times10^{-7}}_{-0.1\times10^{-7}}$ cm^{-3}

$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$

$\Omega_m = 0.27^{+0.04}_{-0.04}$

$\Omega_\nu h^2 < 0.0076$ (95%CL)

$m_\nu < 0.23$ eV (95% CL)

$T_{cmb} = 2.725^{+0.002}_{-0.002}$ K

$n_\gamma = 410.4^{+0.9}_{-0.9}$ cm^{-3}

$\eta = 6.1 \times 10^{-10}{}^{+0.3\times10^{-10}}_{-0.2\times10^{-10}}$

$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$

$\sigma_8 = 0.84^{+0.04}_{-0.04}$ Mpc

$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$

$A = 0.833^{+0.086}_{-0.083}$

$n_s = 0.93^{+0.03}_{-0.03}$

$dn_s/d\ln k = -0.031^{+0.016}_{-0.018}$

$r < 0.71$ (95% CL)

$z_{dec} = 1089^{+1}_{-1}$

$\Delta z_{dec} = 195^{+2}_{-2}$

$h = 0.71^{+0.04}_{-0.03}$

$t_0 = 13.7^{+0.2}_{-0.2}$ Gyr

$t_{dec} = 379^{+8}_{-7}$ kyr

$t_r = 180^{+220}_{-80}$ Myr (95% CL)

$\Delta t_{dec} = 118^{+3}_{-2}$ kyr

$z_{eq} = 3233^{+194}_{-210}$

$\tau = 0.17^{+0.04}_{-0.04}$

$z_r = 20^{+10}_{-9}$ (95% CL)

$\theta_A = 0^\circ.598^{+0.002}_{-0.002}$

$d_A = 14.0^{+0.2}_{-0.3}$ Gpc

$l_A = 301^{+1}_{-1}$

$r_s = 147^{+2}_{-2}$ Mpc

Friedmannien equations

The metric interval is:

FLRW metric is :

$$ds^2 = c^2 dt^2 - a^2(t)\left(dr^2 + f^2(r)\left\{d\theta^2 + \sin^2\theta d\varphi^2\right\}\right)$$

here $f(r)$ is a function which determines global geometrical property of 3D space.

One can put it in the Einstein equations and obtains equation Friedmannien equations which describe the evolution of our Universe.

and three Friedmannien equations are:

$$\frac{1}{2}\dot{a}^2 = \frac{4\pi}{3}Ga^2\rho - \frac{kc^2}{2}$$

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a$$

$$\frac{d\rho}{dt} = -3H(\rho + \frac{p}{c^2})$$

and one more is the equation of state

$$p = q\rho c^2$$

$$H = \frac{1}{a(t)}\frac{da(t)}{dt} \quad \text{is Hubble parameter}$$

and k is cosmological constant which determines the 3D curvature

$$\rho_{crit} = \frac{3H^2}{8\pi G} \quad \text{is critical density}$$

$$\rho_{crit} = 1.88\times10^{-29}\ \frac{g}{cm^3}\left(\frac{H_0}{100\ \text{km/s/Mpc}}\right)^2 = 0.97\times10^{-29}\ \frac{g}{cm^3}$$

$$H_0 = 72\ \text{km/s/Mpc}$$

The Ω parameter

One can introduce the Ω parameter which is more convenient in many cases

$$\Omega = \frac{\rho}{\rho_{crit}}$$

$\Omega > 1$ the closed Universe

$\Omega = 1$ the flat Universe

$\Omega < 1$ the open Universe

$$\frac{kc^2}{2} = \frac{1}{2} H_0^2 a_0^2 (\Omega_0 - 1)$$

Ω_0 is for the present density parameter

Our Universe is filled with different types of matter. Therefore, Ω parameter is sum of several components:

$$\Omega_0 = \Omega_{0m} + \Omega_{0\Lambda} + \Omega_{0r} + \ldots$$

Current observational data from WMAP mission are:

$$\Omega_{tot} = 1.02 \pm 0.02 = \Omega_0$$

$$\Omega_\Lambda = 0.73 \pm 0.04$$

$$\Omega_m = 0.27 \pm 0.04$$

$$\Omega_b = 0.044 \pm 0.004$$

$$\Omega_r = \Omega_\gamma + \Omega_\nu =$$

$$= 1.68 \times \Omega_\gamma = 8.57 \times 10^{-5}$$

$$\rho_{tot}(z) = \frac{3H_0^2}{8\pi G}\left[\Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_r(1+z)^4 + \ldots\right]$$

$$p_{tot}(z) = \frac{3H_0^2 c^2}{8\pi G}\left[-\Omega_\Lambda + \frac{1}{3}\Omega_r(1+z)^4 + \ldots\right]$$

here H_0 is the current Hubble parameter

and $\Omega_m, \Omega_\Lambda, \Omega_r$ are current density parameters

If one traveling back in time $z \rightarrow \infty$

Both density and pressure diverge and tend to infinity. The moment of time which corresponds to infinite value of redshift is called ***singularity***.

The scale factor in singularity

$$a(z) = \frac{a_0}{1+z},$$

tends to zero $\quad a(z) \to 0,$

comoving volume tends to zero $\quad V(z) \to 0.$

Both curvature and tidal forces become infinite.

Cosmological singularity is one of the most intriguing phenomenon in cosmology. The singularity is unavoidable in the General Relativity, and Penrose proof theorem that singularity must emerges.

Newtonian gravity has singularity also, but it is solution of Newtonian equations with measure zero. Exact spherical symmetric distribution of matter collapse to singular state. But small deviation from symmetry destroy this solution and results into nonsingular solution. In \General Relativity independent of initial distribution of matter the result is singularity.

Many authors: Zeldovich and Novikov; Misner, Thorn, Wheleer, and Gorbunov, Rubakov suggest that the solution of singularity problem is connected to Quantum Gravity.

Quantum Gravity does not created yet, but few suggestions can be introduced. Let introduce planckian units. They are constructed from fundamental physical constants

$$m_{Pl} = \sqrt{\frac{\hbar c}{G}} = 2{,}2 \times 10^{-5} \quad \text{g}$$

$$l_{Pl} = \frac{\hbar}{m_{Pl} c} = \sqrt{\frac{\hbar G}{c^3}} = 1{,}6 \times 10^{-33} \quad \text{cm}$$

$$t_{Pl} = \frac{l_{Pl}}{c} = \sqrt{\frac{\hbar G}{c^5}} = 5{,}4 \times 10^{-44} \quad \text{s}$$

Many authors consider them as an edge between classical and quantum gravity. Therefore, one can consider them as starting point for classical description of beginning of the Universe expansion.

These initial values for mass, length, and time correspond to the following values of density, pressure, and temperature:

$$T_{Pl} = 1{,}4 \times 10^{32} \ \ \mathrm{K}$$

$$\rho_{\mathrm{Pl}} = 5{,}3 \times 10^{93} \ \frac{g}{cm^3}$$

$$p_{Pl} = 4{,}8 \times 10^{114} \ \frac{g}{cm \ s^2}$$

Current temperature of the Universe is 3 K. The ratio of planck temperature and current temperature provides us with red shift the creation epoch

$$z \approx 5 \bullet 10^{31}$$

We will refer to the Hubble parameter to describe different epoch during the Universe evolution. Our ansatz is that our Universe is filled with three type of matter: dust like matter with zero pressure, radiation dominated matter with radiation equation of state, and dark energy matter with Lambda term equation of state.

$$H^2(z) = H_0^2\left[\Omega_{0m}(1+z)^3 + \Omega_q + \Omega_{0r}(1+z)^4\right]$$

After the creation of classical space-time the inflationary phase follows. To explain this term one has to show several metaphysical properties of Friedmanien cosmology.

Two main are: flatness problem, horizon problem. A.Guth who invents inflationary theory add also monopole problems, but it is not real problem.

The flatness problem

$$\frac{1}{2}H^2(t)a^2(t)\left(\Omega(t)-1\right)=\frac{1}{2}H_0^2a_0^2\left(\Omega_0-1\right)$$

$$\Omega(z)=1+(1+z)^2\left(\frac{H_0}{H(z)}\right)^2\left(\Omega_0-1\right)$$

$$\frac{\dot{a}^2}{a^2}=H_0^2\left[\Omega_{0m}(1+z)^3+\Omega_q+\Omega_{0r}(1+z)^4\right]$$

At the moment of the Universe creation the Ω parameter has to be as close to unity as

$$\Omega(z_{cr}) = 1 + \frac{\Omega_0 - 1}{\Omega_{0r}} \frac{1}{(1+z)^2} \approx 1 + 8 \bullet 10^{-62}$$

Omega parameter of our Universe has to be ***exactly equal to unity,***

Hypersurface of our Universe must be almost euclidean.

The horizon problem

$$l_a = \frac{l_c}{1+z}$$

$$l_c = \frac{c}{H_0} \int_0^z \frac{dz}{\sqrt{\Omega_{mo}(1+z)^3 + \Omega_q + \Omega_{ro}(1+z)^4}}$$

Angular size of causally connected region at the time of last scattering is 1^o6. The number of causally inconnected regions inside the whole sky sphere is approximately $1.5*10^4$.

All these regions were not in causal connection in the past, they have to have different temperature. But they have almost equal temperature which contradicts to the absence of causal connection.

Both problems are solved in the inflationary theory of the early Universe.

The inflation took place in the very early Universe the state equation is

$$p = -\rho c^2$$

Negative pressure requests the imaginary speed of sound in the media

$$v_s^2 = \frac{\partial p}{\partial \rho} = -c^2$$

It corresponds to strong instability in the media. Therefore this stage finished soon.

During this stage scale factor increase as

$$a(t) = a_{pl}\, \mathrm{e}^{Ht}$$

Difference between kinetical energy and potential energy;

$$\frac{1}{2}\dot{a}^2 - \frac{4\pi}{3}Ga^2\rho = \frac{k}{a^2}$$

tends to zero as $\pm\, \mathrm{e}^{-2H\Delta t}$

Where Δt is duration of inflation stage

If the duration is $H\Delta t = 30$ then the difference between current density and critical density will be of the order of

$$10^{-60}$$

If in the beginning of inflation stage the size of the causal connected volume is of the order of planck size

$$l_{Pl} = \frac{\hbar}{m_{Pl}c} = \sqrt{\frac{\hbar G}{c^3}} = 1{,}6 \times 10^{-33} \text{ cm}$$

At the end it will be

$$l = l_{Pl}\,\mathrm{e}^{30} = 1{,}6 \times 10^{-3}\ \text{cm},$$

and at present epoch it will be of the order of

$$l_0 = 1{,}6 \times 10^{+57}\ \text{cm}.$$

It means that all our observed Universe is small part of causally connected region.

Therefore, independent of value of omega parameter before the inflation at the end of inflation, omega parameter became as close to unity as

$$10^{-60}$$

The size of our Universe to the end of inflation is inflated many order of magnitude. The causally connected region became of the order of 1 cm size. This region pull out by the following expansion. At current moment of time our observed part of the Universe became a small part of this extended region.

That is solution of the metaphysical problem of Friedmanien cosmology

Temperature in the Universe drop down to the end of inflation epoch:

$$T_{end} = T_{pl}\,\frac{a_{pl}}{a_{end}} = 10^{-30} \bullet 10^{32}\ \mathrm{K} \propto 100\,\mathrm{K}$$

After the end of inflation media transform into primordial plasma and reheat to the very high temperature. The red shift of the epoch is large then 10^{20}. Radiation matter dominates into equation of Hubble parameter.

In this moment radiation dominated stage of our Universe started.

The contribution to the expansion rate of the Universe of radiation matter is proportional to 4 power of red shift

$$\Omega_{0r}(1+z)^4$$

While the contribution to the expansion rate of the Universe of dust like matter is proportional to 3 power of red shift

$$\Omega_{0m}(1+z)^3$$

At the red shift *z=3233* contributions equalized and afterwards the expansion rate of the Universe is dominated by dust like matter

$$H^2(z) = H_0^2\left[\Omega_{0m}(1+z)^3 + \Omega_q + \Omega_{0r}(1+z)^4\right]$$

$$\rho = \rho_m + \rho_\Lambda + \rho_r$$

$$p = p_m + p_\Lambda + p_r$$

the evolution of the different

media components runs independently

and $p_m = 0$,

$$p_\Lambda = -\rho_\Lambda c^2$$

$$p_r = \frac{1}{3} \rho_r c^2$$

and three Friedmannien equations are:

$$\frac{1}{2}\dot{a}^2 = \frac{4\pi}{3} G a^2 (\rho_m + \rho_\Lambda + \rho_r)$$

$$\ddot{a} = -\frac{4\pi G}{3}(\rho_m - 2\rho_\Lambda + 2\rho_r)a$$

$$\frac{d(\rho_m + \rho_r)}{dt} = -3H(\rho_m + \frac{4}{3}\rho_r)$$

One can put densities as functions of redshift and rewrite these equations as

$$\rho_m = \rho_{0m}\left(\frac{a_0}{a(t)}\right)^3 \quad \text{and} \quad \rho_m = \rho_{0m}(1+z)^3$$

$$\rho_r = \rho_{0r}\left(\frac{a_0}{a(t)}\right)^4 \quad \text{and} \quad \rho_r = \rho_{0r}(1+z)^4$$

$$\rho_\Lambda = const$$

According to WMAP the total density of our Universe is:

$$\Omega_{total}=1$$

and contribution of different type of matter in density is:

$\Omega_{0m}=0.27$ and $\Omega_q=0.73$

Therefore, the first Friedmannien equation is:

$$\frac{\dot{a}^2}{a^2} = H_0^2\left[\Omega_{0m}(1+z)^3 + \Omega_q + \Omega_{0r}(1+z)^4\right]$$

here Ω_{0r} is present density of the CMBR with respect to critical density

And the second Friedmanien equation becomes

$$\ddot{a} = -\frac{1}{2} H_0^2 \left(\Omega_{0m} (1+z)^3 - 2\Omega_\Lambda + 2\Omega_{0r} (1+z)^4 \right) a$$

and we have two regimes

$$1 + z > \left(\frac{2\Omega_\Lambda}{\Omega_{0m}} \right)^{1/3}$$ is desceleration stage

$$1 + z < \left(\frac{2\Omega_\Lambda}{\Omega_{0m}} \right)^{1/3}$$ is acceleration stage

$z \approx 0.7$

The End